Physics of information technology unit 8 week 5

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Problem 9.1: Twisted Pair and Shielding

Question: Explain why many cables are designed as a twisted pair with a grounded shield.

Solution:

Step 1: **Twisting the Pair:** Twisting causes the two wires to exchange positions repeatedly so that any external electromagnetic interference induces nearly equal noise voltages in both wires. When a differential receiver subtracts the two signals, common-mode noise cancels. (Conceptual)

Step 2: **Shielding:** A grounded shield acts as a Faraday cage, blocking external fields from reaching the conductors and containing the cable's own radiated fields. (Conceptual)

Historical Note: Twisted pair wiring was developed for early telephone systems and is still used in Ethernet cables to minimize noise pickup.

Problem 9.2: Skin Depth in Salt Water

Question: Salt water has a conductivity $\sigma \approx 4\,\mathrm{S/m}$. What is the skin depth at $10^4\,\mathrm{Hz}$?

Solution:

Step 1: The skin depth is given by

$$\delta = \sqrt{\frac{2}{\omega\,\mu\,\sigma}}, \quad \text{(Eq. (8.33))}.$$

Step 2: For $f = 10^4\,\mathrm{Hz}$, compute $\omega = 2\pi f \approx 6.28 \times 10^4\,\mathrm{rad/s}$.

Step 3: Assume $\mu \approx \mu_0 = 4\pi \times 10^{-7}\,\mathrm{H/m}$.

Step 4: Substitute:

$$\delta = \sqrt{\frac{2}{(6.28 \times 10^4)(4\pi \times 10^{-7})(4)}}.$$

Step 5: Compute $\mu_0\sigma = (4\pi \times 10^{-7}) \times 4 \approx 5.03 \times 10^{-6}$.

Step 6: Then, $\omega\,\mu_0\sigma \approx 6.28 \times 10^4 \times 5.03 \times 10^{-6} \approx 0.316$.

Step 7: Finally:

$$\delta = \sqrt{\frac{2}{0.316}} \approx \sqrt{6.33} \approx 2.52\,\mathrm{m}.$$

Conclusion: The skin depth at $10^4\,\mathrm{Hz}$ in salt water is approximately 2.5 meters.

Problem 9.3: Power Flow in a Coaxial Cable via the Poynting Vector

Question: Integrate the Poynting vector $\mathbf{P} = \mathbf{E} \times \mathbf{H}$ over the cross-sectional area of a coaxial cable and relate it to the current and voltage.

Solution:

Step 1: Electric Field: The voltage is defined by

$$V = -\int \mathbf{E} \cdot d\mathbf{l} \quad \text{(Eq. (8.1))}.$$

By symmetry, the radial electric field is

$$E(r) = \frac{V}{r \ln(r_o/r_i)}.$$

Step 2: Magnetic Field: Using Ampère's Law (Eq. (8.41)), the magnetic field in the region is

$$H(r) = \frac{I}{2\pi r}.$$

Step 3: Poynting Vector: The magnitude is

$$S(r) = E(r)H(r) = \frac{V}{r \ln(r_o/r_i)} \cdot \frac{I}{2\pi r} = \frac{VI}{2\pi \ln(r_o/r_i)} \frac{1}{r^2}.$$

Step 4: Total Power: The differential area is $dA = 2\pi r\, dr$. Thus,

$$P = \int_{r_i}^{r_o} S(r)\, dA = \frac{VI}{\ln(r_o/r_i)} \int_{r_i}^{r_o} \frac{1}{r}\, dr.$$

Step 5: Evaluate the Integral:

$$\int_{r_i}^{r_o} \frac{1}{r}\, dr = \ln\left(\frac{r_o}{r_i}\right).$$

Step 6: Result:

$$P = VI.$$

Conclusion: The integrated power equals $P = VI$, which is the standard power relationship.

Problem 9.4: Characteristic Impedance and Signal Velocity for Two Parallel Strips

Question: For a transmission line consisting of two parallel strips of width w and separated by distance h, find the characteristic impedance Z_0 and the signal velocity v (ignoring fringing fields).

Solution:

Step 1: Capacitance per Unit Length: Approximating the strips as parallel plates,

$$C = \epsilon \frac{w}{h} \quad \text{(analogous to Eq. (8.16))}.$$

Step 2: Inductance per Unit Length: Similarly,

$$L = \mu \frac{h}{w} \quad \text{(from the discussion around Eq. (8.43)).}$$

Step 3: Characteristic Impedance:

$$Z_0 = \sqrt{\frac{L}{C}} = \frac{h}{w}\sqrt{\frac{\mu}{\epsilon}}.$$

Step 4: Signal Velocity:

$$v = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\mu\epsilon}},$$

which is the speed of light in the medium.

Conclusion:

$$\boxed{Z_0 = \frac{h}{w}\sqrt{\frac{\mu}{\epsilon}}, \quad v = \frac{1}{\sqrt{\mu\epsilon}}.}$$

Problem 9.5: Coaxial Cable RG58/U

RG58/U coaxial cable has:

- Relative permittivity $\epsilon_r = 2.26$,
- Inner radius $r_i = 0.406\,\text{mm} = 0.406 \times 10^{-3}\,\text{m}$,
- Outer radius $r_o = 1.48\,\text{mm} = 1.48 \times 10^{-3}\,\text{m}$.

(a) Characteristic Impedance: A common formula for coaxial cable is

$$Z_0 = \frac{60}{\sqrt{\epsilon_r}} \ln\left(\frac{r_o}{r_i}\right) \quad \text{(Eq. (8.45)).}$$

Compute:

$$\sqrt{2.26} \approx 1.503, \quad \ln\left(\frac{1.48 \times 10^{-3}}{0.406 \times 10^{-3}}\right) \approx \ln(3.645) \approx 1.293.$$

Thus,

$$Z_0 \approx \frac{60}{1.503} \times 1.293 \approx 39.92 \times 1.293 \approx 51.6\,\Omega.$$

(b) Signal Velocity: The velocity in a coaxial cable is given by

$$v = \frac{c}{\sqrt{\epsilon_r}}, \quad \text{(from wave propagation, Eq. (8.57)).}$$

With $c \approx 3 \times 10^8\,\text{m/s}$:

$$v \approx \frac{3 \times 10^8}{1.503} \approx 2.0 \times 10^8\,\text{m/s}.$$

(c) Maximum Cable Length for a 1 ns Clock Cycle: Assuming a propagation delay of 4.6 ns/m, then for a 1 ns delay:

$$L \approx \frac{1\,\text{ns}}{4.6\,\text{ns/m}} \approx 0.217\,\text{m}.$$

(Note: This is one interpretation; other approaches might consider the total pulse width.)

(d) Matching Impedance with Thinner Cable: For a cable with an outer diameter of 30 mils (1 mil = 0.001 in; 1 in = 0.0254 m):

$$0.03\,\text{in} \approx 0.03 \times 0.0254 = 0.000762\,\text{m},$$

so the outer radius is

$$r_o \approx 0.000762/2 \approx 0.000381\,\text{m}.$$

To maintain $Z_0 \approx 50\,\Omega$ with $\epsilon_r = 2.26$, we use:

$$\ln\left(\frac{r_o}{r_i}\right) = \frac{50 \times 1.503}{60} \approx 1.253.$$

Thus,

$$\frac{0.000381}{r_i} = e^{1.253} \approx 3.5 \quad \Longrightarrow \quad r_i \approx \frac{0.000381}{3.5} \approx 0.000109\,\text{m}.$$

The inner diameter is about $2r_i \approx 0.000218\,\text{m}$ (0.218 mm).

(e) Frequency Where Wavelength $\approx$ Cable Diameter: For RG58/U, the outer diameter is $\approx 1.48 \times 10^{-3}\,\text{m}$. The wavelength is

$$\lambda = \frac{v}{f} = \frac{c}{\sqrt{\epsilon_r}\,f}.$$

Setting $\lambda \approx 1.48 \times 10^{-3}\,\text{m}$:

$$f \approx \frac{3 \times 10^8}{1.503 \times 1.48 \times 10^{-3}} \approx 1.35 \times 10^{11}\,\text{Hz}.$$

Thus, about 135 GHz.

Problem 9.6: CAT6 Cable and Reflections at a T Connector

CAT6 twisted pair cable has a propagation delay of 4.6 ns/m and an impedance of 100 ohm.

(a) Physical Length of a 64-Byte Frame: A 64-byte frame has $64 \times 8 = 512$ bits. At 1 Gbps, the frame duration is:

$$T = \frac{512}{10^9} = 512\,\text{ns}.$$

Thus, the cable length is:

$$L = \frac{512\,\text{ns}}{4.6\,\text{ns/m}} \approx 111\,\text{m}.$$

(b) Reflection Coefficient at a T Connector: When a T connector splits one cable into two, the effective load impedance becomes:

$$Z_L = \frac{100\,\Omega}{2} = 50\,\Omega.$$

The reflection coefficient is defined as:

$$R = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{50 - 100}{50 + 100} = \frac{-50}{150} \approx -0.33.$$

This indicates that approximately 33% of the voltage is reflected, with a phase inversion.